

1) A 2 kg mass is attached to a spring with constant $k = 50 \text{ N/m}$ and placed on a smooth frictionless surface as shown in Figure 1.1.

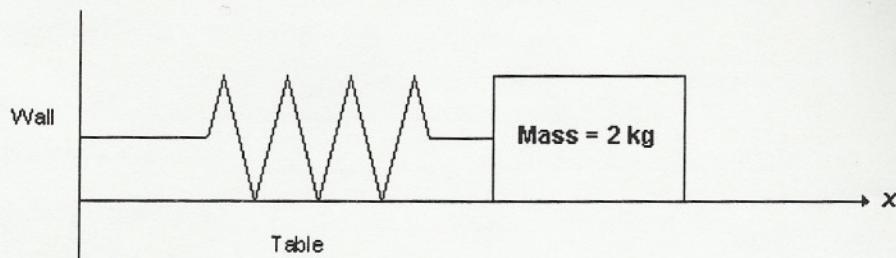


Figure 1.1

a) If the spring is displaced 4.8 cm from its equilibrium position and then released at time $t = 0$ seconds:

- How much potential energy is in the system at time $t = 0$ seconds? ~~0.0576 J~~ (2 marks) 2
- How much kinetic energy is in the system at time $t = 0$ seconds? ~~0 J~~ (2 marks) 2
- How much total energy is in the system at time $t = 4.2$ seconds? ~~0.0345 J~~ (2 marks) 0

b) A second experiment is performed with this system. It was seen that at time $t = 0$ seconds the mass had a velocity of 3 cm/s in the positive x direction and was at position $x = 2 \text{ cm}$:

- What is the total energy of this system at time $t = 12.1$ seconds? ~~9.52 x 10^-3 J~~ (2 marks) 0

If the motion of the mass is described by the equation $x(t) = A \cos(\omega t + \phi_0)$ what are:

- ω ? ~~5 rad/s~~ (1 mark) 0
- A ? ~~0.42 m~~ (2 marks) 0
- ϕ_0 ? ~~X~~ (2 marks) 0

c) Now assume that the spring is changed to one of unknown spring constant. However, you do know that hanging a 2 kg mass from the new spring stretches it 10 cm from its stable equilibrium position.

i) What is the total energy in the new system if at $t = 4$ seconds the mass was at position $x = 2.5 \text{ cm}$ and it was moving to the left with a velocity of 4 cm/s? ~~X~~ (3 marks) 0

$$\text{I) } PE = \frac{1}{2} k s^2$$

$$PE = \frac{1}{2} (50) (0.048)^2$$

$$PE = \boxed{0.0576 \text{ J}}$$

$$\text{II) } KE = \frac{1}{2} m v^2$$

$$KE = \frac{1}{2} m (0)^2$$

$$KE = \boxed{0 \text{ J}}$$

$$x = A \cos(\omega t + \phi_0)$$

$$x = (4.8) \cos[(5)(4.2) + 0]$$

$$x = -2.63 \text{ cm} = -0.0263 \text{ m}$$

$$\text{III) } E = K + U$$

$$E = \frac{1}{2} m v^2 + \frac{1}{2} k s^2$$

$$E = \frac{1}{2} (2) (0.131)^2 + \frac{1}{2} (50) (-0.0263)^2$$

$$E = 0.0472 + 0.0173$$

$$E = \boxed{0.0645 \text{ J}}$$

$$\left. \begin{array}{l} \omega = \sqrt{\frac{k}{m}} \\ \omega = \sqrt{\frac{50}{2}} \\ \omega = 5 \text{ rad/s} \end{array} \right\}$$

$$v = -\omega A \cos(\omega t + \phi_0)$$

$$v = (-5)(0.048) \cos[(5)(4.2) + 0]$$

$$v = 0.131 \text{ m/s}$$

$$v = -\omega A \cos(\omega t + \phi_0)$$

$$E = \frac{1}{2} m v^2 + \frac{1}{2} k s^2$$

$$E = \frac{1}{2} (2) (0.131)^2 + \frac{1}{2} (50) (-0.0263)^2$$

A nylon guitar string has a linear mass density of 7.2 g/m and is under a tension of 150 N . The fixed supports are 75 cm apart. It is observed that the string is oscillating with the four node standing wave pattern seen in Figure 2.1. Answer the following questions with respect to the two traveling waves that are required to produce this superposition or standing wave.

i) What are their speeds? 4.56 m/s ~~2~~ (2 marks)

ii) What are their wavelengths? 50 cm ~~2~~ (2 marks)

iii) What are their frequencies? 9.12 Hz ~~X~~ (2 marks)

iv) What are their amplitudes? 0.0798 m ~~2~~ (2 marks)

v) What are their maximum transverse speeds? 0.726 m/s ~~X~~ (3 marks) ~~2~~ (2 marks)

vi) What would their frequency be if the standing wave pattern on the same guitar string had 7 nodes? 18.2 Hz ~~X~~ (3 marks) ~~2~~ (2 marks)

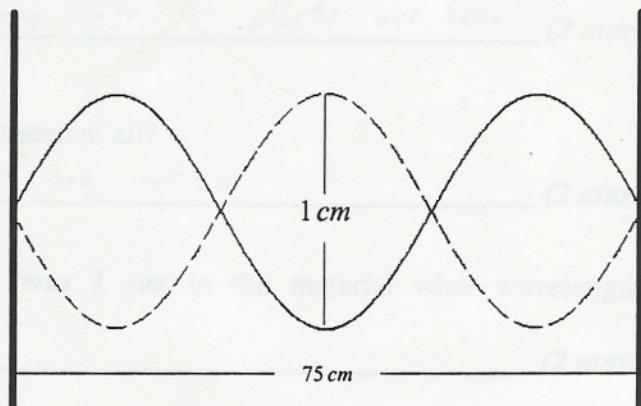


Figure 2.1

$$I) V = \sqrt{\frac{T}{\mu}}$$

$$V = \sqrt{\frac{150 \text{ N}}{7.2 \text{ g/m}}}$$

$$V = \boxed{4.56 \text{ m/s}}$$

$$II) \lambda = \frac{2L}{n}$$

$$\lambda = \frac{2(75 \text{ cm})}{3}$$

$$\lambda = \boxed{50 \text{ cm}} = 0.50 \text{ m}$$

$$III) V = f \lambda$$

$$f = \frac{V}{\lambda}$$

$$f = \frac{4.56 \text{ m/s}}{0.50 \text{ m}}$$

$$f = \boxed{9.12 \text{ Hz}}$$

$$IV) \lambda = \frac{2L}{n}$$

$$\lambda = \frac{2(75 \text{ cm})}{6}$$

$$\lambda = \boxed{25 \text{ cm}}$$

$$\lambda = 0.25 \text{ m}$$

$$V = f \lambda$$

$$f = \frac{V}{\lambda}$$

$$f = \frac{4.56 \text{ m/s}}{0.25 \text{ m}}$$

$$f = 18.2 \text{ Hz}$$

$$VI) \omega = \frac{2\pi}{T}$$

$$\omega = \frac{2\pi}{0.110 \text{ s}}$$

$$\omega = 57.1 \text{ rad/s}$$

$$k = \frac{2\pi}{\lambda}$$

$$k = \frac{2\pi}{0.50 \text{ m}}$$

$$k = 12.6 \text{ rad/m}$$

$$T = \frac{1}{f}$$

$$T = \frac{1}{9.12 \text{ Hz}}$$

$$V = \frac{A}{T}$$

$$T = \frac{2\pi A}{V}$$

$$A = \boxed{TV}$$

3) Light that contains electromagnetic waves over the wavelength range from 450 nm to 600 nm goes through a very small hole such that what comes out behaves like a single circular source. Assume there is a flat $3 \mu\text{m}$ thick piece of $n = 1.3$ plastic a large distance away from the circular source such that when the light gets to the plastic the wavefronts can be considered parallel and traveling in a direction that is normal to the surface of the plastic. You can assume that air surrounds all of the components in this system. Also assume the coherence length of the light in the material is greater than $10 \mu\text{m}$.

a) If the plastic is viewed with the reflected light:

i) What wavelengths are seen in fully constructive interference?

~~600 nm~~ ~~500 nm~~

(2 marks)



ii) What wavelengths are not seen at all?

~~545 nm~~ ~~462 nm~~

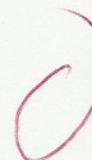
(2 marks)



b) If the plastic is viewed with the transmitted light:

i) What wavelengths are seen in fully constructive interference?

~~Any wavelength going into the plastic are seen~~ (2 marks)



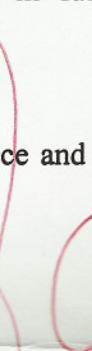
ii) What wavelengths are not seen at all?

~~No wavelengths are not seen~~ (2 marks)



iii) If the coherence length was $1 \mu\text{m}$ in the material what wavelengths are in fully constructive interference?

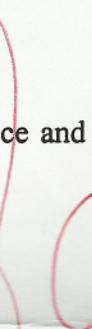
(2 marks)



c) Now if we put a piece of $n = 1.35$ glass on the back of the original plastic piece and if the plastic is viewed with the reflected light:

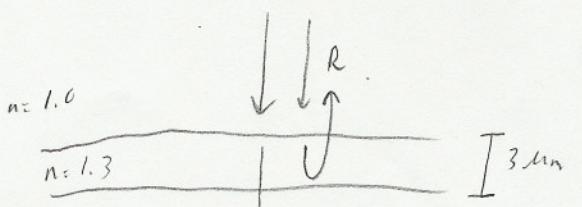
i) What wavelengths are seen in fully constructive interference?

(2 marks)



ii) What wavelengths are not seen at all?

(2 marks)



~~II) $m + \frac{1}{2} = \frac{d}{\lambda}$~~

$$\lambda = \frac{3000 \text{ nm}}{m + \frac{1}{2}}$$

$m=0 \rightarrow \lambda = 4000 \text{ nm}$
 $m=1 \rightarrow \lambda = 2000 \text{ nm}$
 $m=2 \rightarrow \lambda = 1200 \text{ nm}$
 $m=3 \rightarrow \lambda = 857.1 \text{ nm}$
 $m=4 \rightarrow \lambda = 667 \text{ nm}$
 $m=5 \rightarrow \lambda = 545 \text{ nm}$
 $m=6 \rightarrow \lambda = 462 \text{ nm}$

A) I) $m = \frac{d}{\lambda}$

$$\lambda = \frac{d}{m}$$

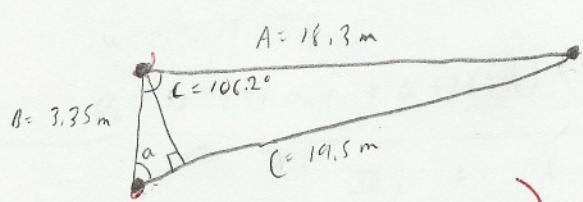
$$\lambda = \frac{3 \times 10^{-6} \text{ m}}{m}$$

$m=1 \rightarrow \lambda = 3000 \text{ nm}$
 $m=2 \rightarrow \lambda = 1500 \text{ nm}$
 $m=3 \rightarrow \lambda = 1000 \text{ nm}$
 $m=4 \rightarrow \lambda = 750 \text{ nm}$
 $m=5 \rightarrow \lambda = 600 \text{ nm}$
 $m=6 \rightarrow \lambda = 500 \text{ nm}$
 $m=7 \rightarrow \lambda = 428.6 \text{ nm}$

B) Any wavelengths going into the plastic are transmitted, and therefore, are seen.

4) A distance of 3.35 m separates two loudspeakers on an outdoor stage. A listener is 18.3 m from one of the speakers and 19.5 m from the other. During the sound check, a signal generator drives the two speakers in phase with the same amplitude and frequency. Assume there is no significant attenuation of the amplitude as the wave propagates from the source to the listener. The transmitted frequency is swept through the audible range from 20 Hz to 20 kHz . In air, the speed of sound is 330 m/s and the speed of light is $3.0 \times 10^8 \text{ m/s}$

- What are the three lowest frequencies at which the listener will hear no signal due to destructive interference? 341 Hz 569 Hz 797 Hz (4 marks) ~~-2~~
- What are the three lowest frequencies at which the listener will hear a maximum signal due to constructive interference? 228 Hz 455 Hz 683 Hz (4 marks) ~~-2~~
- While listening to the sound check the person is also watching a nearby baseball game. He sees the batter swing the bat and 2.85 seconds later he hears the "crack" of the bat as the batter hits the ball. How far is the batter away from the listener? 941 m (2 marks)
- If the listener moves to where the batter hit the ball and he does not hear any of the 200 Hz signal at what possible angle θ , with respect to the perpendicular bisector of the line that joins the speakers, could he now be standing? $79.3^\circ = 0.44 \pi \text{ rad}$ (3 marks) ~~-3~~



$$c^2 = A^2 + B^2 - 2AB (\cos \alpha)$$

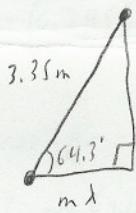
$$\cos \alpha = \frac{c^2 - A^2 - B^2}{-2AB}$$

$$\cos \alpha = -0.278$$

$$\alpha = 106.2^\circ$$

$$A \cos(K\tau - \omega t) \rightarrow$$

~~Graph~~



$$\frac{\sin \alpha}{A} = \frac{\sin C}{c}$$

$$\sin \alpha = \frac{\sin (106.2^\circ)}{19.5 \text{ m}} (18.3 \text{ m})$$

$$(3.35 \text{ m}) (\cos 64.3^\circ) = \lambda$$

$$1.45 \text{ m} = \lambda$$

$$\lambda = \frac{1.45}{\text{m}}$$

$$\sin \alpha = 0.901$$

$$\alpha = 64.3^\circ$$

$$\lambda = \frac{1.45 \text{ m}}{m \frac{1}{\lambda}} \left\{ \begin{array}{l} m=1 \rightarrow \lambda = 0.967 \text{ m} \\ m=2 \rightarrow \lambda = 0.58 \text{ m} \\ m=3 \rightarrow \lambda = 0.414 \text{ m} \end{array} \right.$$

$$V = f \lambda$$

$$f = \frac{V}{\lambda}$$

$$f = \frac{330 \text{ m/s}}{\lambda}$$

$$\lambda = 0.967 \rightarrow f = 341 \text{ Hz}$$

$$\lambda = 0.58 \rightarrow f = 569 \text{ Hz}$$

$$\lambda = 0.414 \rightarrow f = 797 \text{ Hz}$$

$$\lambda = \frac{1.45 \text{ m}}{m \frac{1}{\lambda}} \left\{ \begin{array}{l} m=1 \rightarrow \lambda = 1.45 \text{ m} \\ m=2 \rightarrow \lambda = 0.725 \text{ m} \\ m=3 \rightarrow 0.483 \text{ m} \end{array} \right.$$

$$f = \frac{330 \text{ m/s}}{\lambda} \left\{ \begin{array}{l} \lambda = 1.45 \text{ m} \rightarrow f = 228 \text{ Hz} \\ \lambda = 0.725 \text{ m} \rightarrow f = 455 \text{ Hz} \\ \lambda = 0.483 \text{ m} \rightarrow f = 683 \text{ Hz} \end{array} \right.$$

5) A wave on a long string is described by $\bar{y}(x, t) = 0.085 \cos[21.3(x + 50.7 - 2.1t)] \text{ m}$ where x and t are in metres and seconds respectively.

i) What is the velocity of the wave? 2.1 m/s (2 marks)

ii) What is the wavelength of the wave? 0.295 m (2 marks)

iii) What is the frequency (in Hz) of the wave? 7.12 Hz (2 marks)

iv) What is the maximum transverse velocity of the string? 0.605 m/s (2 marks)

v) What wave would be required to produce complete destructive interference everywhere when superimposed with this wave? $y = [0.085 \cos(21.3x - 44.7t + 2.75\pi)] \text{ m}$ (2 marks)

vi) Express the phase constant of this wave as an angle in radians that is in between 0 and 2π .

$1.75\pi \text{ radians}$ (2 marks)

$$y = A \cos(kx - \omega t + \phi_0)$$

$$A = 0.085 \text{ m}$$

$$k = 21.3 \text{ rad/m}$$

$$\omega = 44.7 \text{ rad/s}$$

$$\phi_0 = 1079.91 \text{ rad} = 0.87(2\pi) = 1.75\pi$$

$$\text{II) } \lambda = \frac{2\pi}{k}$$

$$\lambda = \frac{2\pi}{21.3}$$

$$\lambda = [0.295 \text{ m}]$$

$$\text{III) } v = f\lambda$$

$$f = \frac{v}{\lambda}$$

$$f = \frac{21.3 \text{ m/s}}{0.295 \text{ m}}$$

$$f = [7.12 \text{ Hz}]$$

$$\text{IV) } T = \frac{1}{f}$$

$$T = [1.412 \text{ Hz}]$$

$$T = 0.141 \text{ s}$$

$$v = \frac{A}{T}$$

$$v = \frac{0.085 \text{ m}}{0.141 \text{ s}}$$

$$v = [0.605 \text{ m/s}]$$

$$y_1 = A \cos(kx - \omega t + \phi_0 + \pi)$$

$$y_2 = 0.085 \cos(21.3x - 44.7t + 1.75\pi + \pi)$$

$$y = 0.085 \cos(21.3x - 44.7t + 2.75\pi)$$

$$\text{VI) } \phi_0 = 1079.91 \text{ rad} = 343.75\pi$$

$$\frac{343.75\pi}{2\pi} = 171.87 \text{ rotations}$$

$$171.87 - 171 = 0.87$$

$$(0.87)(2\pi) = [1.75\pi]$$

6) Assume a plane wave is incident at an angle β upon a barrier with a small opening of width $a = 2 \mu m$. The angle of incidence is as shown in Figure 6.1. Also assume that air surrounds all of the components of this system.

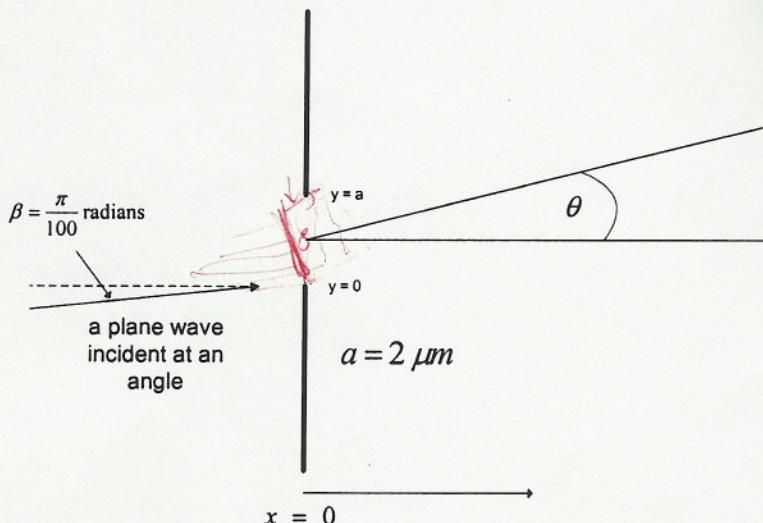


Figure 6.1

a)

- What is an equation that describes the phase difference between the wavefronts when they hit the barrier at $y = 0$ and $y = a$? -2 (2 marks)
- At what angle θ are all wavelengths fully constructive? -2 (2 marks)
- At what angles θ are the first diffraction minima if the wave is $\lambda = 510 nm$ light?
 $14.8^\circ = 0.082\pi \text{ rad}$ -2 no + go + e (3 marks)
- With this setup how far apart in angle are the $m = 1$ diffraction minima for light of wavelengths $510 nm$ and $520 nm$? Please express your angle in radians. $0.0837\pi \text{ rad}$ -2 (3 marks)

b)

- Now assume the $510 nm$ light is normally incident and a second identical slit is placed $d = 10 \mu m$ away, at what angle would you find the first missing interference maximum?
 $2.92^\circ = 0.0162\pi \text{ rad}$ -2 (2 marks)
- In this new configuration if the left hand side of the second slit is covered with a $0.25 \mu m$ thick piece of $n = 1.2$ plastic what is the phase difference between the wavefronts of the $510 nm$ light at the right hand side of each of the slit openings? -2 (2 marks)

A) III) $\sin \theta = m \frac{\lambda}{a}$

$$\sin \theta = (1) \frac{(510 \times 10^{-9} \text{ m})}{2 \times 10^{-6} \text{ m}}$$

$$\sin \theta = 0.255$$

$$\theta = 14.8^\circ$$

A) IV) $\sin \theta = m \frac{\lambda}{a}$

$$\sin \theta = (1) \frac{(520 \text{ nm})}{200 \text{ nm}}$$

$$\sin \theta = 0.26$$

$$\theta = 15.1^\circ$$

$$\theta = 0.0837\pi \text{ rad}$$

C) $m_i = x \frac{m_i}{d}$

$$\sin \theta = m_i \left(\frac{\lambda}{d} \right)$$

B) III)